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Sub-picosecond Electron Bunch Diagnostic

Steven J. Russell, Bruce E. Carlsten*, Dinh C. Nguyen, John Kinross-Wright

Abstract

This is the final report of a three-year, Laboratory-Directed Research and Development (LDRD) project at the Los Alamos National Laboratory (LANL). The purpose of this LDRD project was to develop new, inexpensive, simple electron beam bunch-length diagnostics with resolutions below ¼ picosecond (ps). Our approach to this problem was three pronged. First, it was our intent to build a streak camera with ½ ps resolution. This conventional diagnostic was to be used to verify the results of two novel measurement techniques. Second, we wished to investigate the utility of off-axis coherent Smith-Purcell radiation as a bunch-length diagnostic. Finally, we planned to demonstrate a measurement of the electron bunch-length using a transversely deflecting RF field and a beam position monitor (BPM).

Background and Research Objectives

The generation of very short bunch lengths (temporal duration of 1 ps or less) is a thrust area in the development of high-brightness electron beams for accelerator applications. Sub-picosecond physics, chemistry, and materials science are rapidly growing fields that are of great importance to the scientific community. Ultra-fast phenomena that require sub-picosecond radiation pulses include the excitation, ionization, and relaxation of atoms and molecules and beam-induced quantum effects. It is important in the investigation of these phenomena that the ultra-fast radiation source be tunable. Tunable, sub-picosecond radiation sources (such as free-electron lasers and synchrotrons) require sub-picosecond electron bunches to drive them.

The need for sub-picosecond electron sources has stimulated the development of sub-picosecond electron bunch compression technology. Results from experiments in recent years include compressed full-width-at-half-maximum (FWHM) bunch lengths of about 100 femtosecond (fs) (bunch charge of 0.01 nC) at Stanford University [1], of 700 fs (bunch charge of 0.1 nC) at the University of Tokyo [2], and of 250 fs (bunch charge of 0.1 nC) and of 600 fs (bunch charge of 1 nC) here at LANL [3]. However, the development of diagnostic techniques with sub-picosecond resolution has not kept pace

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with the development of sub-picosecond electron beams. The key obstacle to the further evolution of short bunch technology and its applications is the absence of non-intercepting diagnostic techniques that can measure bunch-lengths reliably and inexpensively below 500 fs.

There are several state-of-the-art sub-picosecond beam measurements that have been demonstrated and are used on a somewhat routine basis. However, these tend to be very expensive, costing between $100k and $250k, are often complicated to implement and usually intercept the beam.

The most direct diagnostic of the beam bunch-length is to use a conventional streak camera to measure the temporal distribution of optical transition radiation (OTR) light [4] or Cherenkov radiation [5]. These radiations are generated by directing the electron beam onto a screen. Streak cameras capable of measuring FWHM bunch lengths down to 200 fs are available and have been used in experimental situations [6]. The main drawbacks to this technique are the exorbitant cost of a 200 fs streak camera and the fact that the beam is effectively destroyed when it impacts the screen. Also, the temporal resolution of streak camera technology is essentially limited to 200 fs by time dispersion within the camera itself, offering little chance of substantial improvement in this technique's resolution. This measurement technique was employed in reference 2.

Another popular bunch-length measurement is to use a Michelson interferometer to analyze the OTR light and deduce the beam bunch-length from the fringe pattern of the light. Like the streak camera method, this diagnostic is very straightforward. However, the beam is also destroyed with this technique and its implementation is expensive. Additionally, in order to avoid dispersion that is on the order of the time scales one is trying to measure, the optical layout of such a system is very complicated. This measurement technique was employed in reference 1.

Other types of intercepting bunch-length measurements that have been demonstrated to date include inferring/measuring the bunch-length or tomographically reproducing the temporal distribution of the bunch by observing the energy spread of the bunch in a spectrometer as some accelerator parameter is varied [3,7,8]. These techniques are inexpensive, but inferring the bunch length is prone to errors [3], tomographic reconstruction [8] requires a level of beam stability that cannot always be achieved and the very accurate zero-phasing method described in [7] makes assumptions that are not valid in all circumstances. Also, like the methods already described, these cannot be conducted in parallel with other experiments.

In recent years a measurement technique that does not intercept the electron beam has also been demonstrated. It uses the coherent synchrotron radiation that is generated by the beam as it travels through a bend (alternatively, the beam could be intercepted and the OTR light used instead) [9]. The radiation emitted at wavelengths long compared to the bunch length is coherent and scales as the number of particles N within the bunch squared (N^2); the radiation emitted at wavelengths short compared to the bunch length is incoherent and scales only with N. Since N is on the order of 10^{10} for typical bunches, the coherent radiation is very much larger than the incoherent radiation. By observing at what wavelength the transition from coherent radiation to incoherent radiation occurs, one can determine the rms bunch-length of the electron beam. In addition, features of the radiation spectrum can in principle be used to determine characteristics of the bunch's temporal distribution. However, this measurement is very expensive. A polarizing grid, Michelson spectrometer, which measures the radiation spectrum, is required to implement this diagnostic and this piece of equipment alone costs well over $100k.

It should be clear that there are state-of-the-art techniques capable of measuring the bunch-length of an electron beam with sub-picosecond resolution. What should also be clear, however, is that these diagnostics have one or more of the following drawbacks: they are expensive, they are complicated to implement or they destroy the beam. Our goal was spur the creation of one or more simple, inexpensive sub-picosecond beam-bunch diagnostics that could be used on a routine basis. The ultimate end would be to see the development of a beam-bunch measurement that would be as routine and as simple to implement as a current or beam position measurement.

In this LDRD we proposed to investigate the efficacy of two novel, inexpensive, non-intercepting beam bunch diagnostics. The first is based on coherent Smith-Purcell radiation and the second utilizes a transversely deflecting RF cavity and a simple beam position monitor. As part of this investigation, we planned to build a ½ ps streak camera so that we could verify any results produced by the two novel concepts.

Importance to LANL's Science and Technology Base and National R&D Needs

LANL has a unique, high-charge, sub-picosecond electron injector, the Sub-picosecond Accelerator (SPA). A low energy bunch compressor [10] was installed on this beam line (funded by the Laboratory LDRD program), and has demonstrated compressed FWHM bunch lengths of 250 fs at low charge (0.1 nC) and 600 fs at high charge (1 nC) [3,10]. At the time of its construction, SPA was the only injector capable of producing sub-picosecond, 1 nC electron bunches in the world and established LANL at the forefront of this technology.

There are two large projects currently under development in this country that will benefit from the work that LANL is doing in this field: the Next Linear Collider (NLC)

[11] and the Linear Coherent Light Source (LCLS) [12]. Both require improvements in state-of-the-art electron beam bunches. For this work to proceed and attain the required level of technological maturity, sub-picosecond electron beam-bunch diagnostics must also progress.

Scientific Approach and Accomplishments

500 fs calibration tool

Our approach to this LDRD project was three pronged. First, it was our intent to build the equivalent of a 500 fs streak camera tailored to the SPA machine. This device would be used to benchmark our two novel diagnostic ideas. The heart of the streak camera was to be a Femtochron 25 streak tube from Photek Limited and an intensified CCD array. These two components cost about \$35K and \$20K respectively (much cheaper than a complete, commercial streak camera). The streak and gating electronics were to be built in-house. The finished device was to be mounted on an assembly appropriate for use on the SPA. In addition, a special OTR screen assembly and light collecting optical system was to be built for maximizing the light signal entering the camera so that single pulse measurements could be made.

The coherent Smith-Purcell radiation concept

The first of our two novel bunch-length diagnostics is based on coherent Smith-Purcell radiation (SPR) [13,14,15,16,17]. When an electron beam traverses close to the surface of a metallic diffraction grating, SPR is emitted (Figure 1). The well known dispersion relation for SPR is (Figure 2)

$$\lambda = \frac{d}{n}\left(\frac{1}{\beta} - \cos\theta\right)$$

where d is the grating period, n is the diffraction order, β is the velocity of the beam divided by the speed of light and θ is the angle of the emitted SPR with respect to the beam propagation direction. The intensity of this radiation per unit solid angle per unit grating length as a function of emitted angle is given, by [18]:

$$\frac{dP_n}{d\Omega} = \frac{eIn^2L\beta^3}{2\varepsilon_0 d^2}\left|R_n\right|^2\left(1 + Nf\left(\sigma_z, \lambda, \theta\right)\right)\frac{\sin^2(\theta)\cos^2(\phi)}{\left[1 - \beta\cos(\theta)\right]^3}$$
$$\times e^{-\left(4\pi h/\beta\gamma\lambda\right)\sqrt{1+(\beta\gamma\sin\theta\sin\phi)^2}}, \qquad (1)$$

where e is the electron charge, I is the beam current, n is the grating order, L is the total length of the grating, ε_0 is the permitivity of free space, $|R_n|^2$ is the square of the grating reflectivity, N is the number of electrons in the bunch, θ and ϕ are the angles of observation with respect to the beam direction, h is the beam height above the grating, and $f(\sigma_z, \lambda, \theta)$ is a form factor between 0 and 1 given by the Fourier transform of the longitudinal density function of the beam, $S(z)$:

$$f(\sigma_z, \lambda, \theta) = \left| \int S(z) e^{i2\pi z \cos\theta / \lambda} dz \right|^2 .$$

At wavelengths short compared to the beam bunch-length the SPR is incoherent and the form factor $f(\sigma_z, \lambda, \theta)$ is approximately zero. When the beam-bunch is comparable to, or less than, the wavelength, the SPR is coherent and the form factor approaches 1. Since the number of electrons in an electron bunch is typically very large, on the order of 10^{10}, the coherent signal is much more intense than the incoherent signal.

Figure 3 is a plot of the intensity (equation 1) versus θ for SPR radiation in the x plane ($\phi = 0$) for three electron beam bunch-lengths: 2 ps, 1 ps and ½ ps. The energy of the electron beam is 8 MeV, the groove spacing, d, is 1 mm, n is equal to 1 and the beam height above the grating, h, is 0.5 mm. The three peaks are from coherent SPR. The incoherent radiation peak occurs at around 30 degrees and is a factor of approximated 10^7 down from the coherent peaks. We propose that we can determine bunch-length of the beam from either the angular position or the intensity of its coherent SPR peak.

What is especially attractive about this diagnostic is that it scales naturally to shorter time scales. By reducing the groove spacing of the grating, we in turn reduce the resolution of the measurement technique. Figure 4 is a plot of the SPR intensity versus θ for the same beam energy, but with a groove spacing of 0.1 mm, a beam height of 0.1 mm and electron beam bunch-lengths of 200 fs, 100 fs and 50 fs. By a trivial change in our grating period, we have gained a factor of 10 in resolution.

The transversely deflecting RF field and beam position monitor concept

A beam position monitor (BPM) has four electric field probes located at 90° intervals around a section of beam pipe at a single axial position. As the electron travels through the aperture of the BPM, the probes sense the image charge that the beam induces in the metal walls of the beam pipe. By combining the four signals appropriately, one can measure the position of the beam (first moment) within the BPM and the value of the second moment

$$\langle x^2 \rangle - \langle y^2 \rangle ,$$

where the angled brackets indicate and average over the beam distribution (Figure 5) [16,17].

If the beam is passed through a transversely deflecting RF cavity, a correlation between the axial dimension of the bunch and the horizontal or vertical dimension can be induced. By phasing this cavity correctly, the center of the beam-bunch is not deflected, but the head and tail of the bunch are kicked in opposite directions. As the bunch drifts to the BPM the bunch will spread in the chosen transverse plane (Figure 6). Since this increase only occurs in $\langle x^2 \rangle$ or $\langle y^2 \rangle$, measuring the second moment with the BPM is equivalent to measuring the bunch-length[1]. Also, since this spread is correlated, it can be removed by a second RF cavity downstream with little degradation of beam quality.

This type of measurement has been used before, but with a screen and a camera to measure the change in beam size instead of the BPM [3]. The advantage one gains with the BPM is that the beam can be allowed to drift indefinitely, as long as it does not intercept the beam pipe walls, with a corresponding increase in the resolution of the measurement. Also, the BPM is non-intercepting and preserves the beam quality. We expect that a resolution of about 500 fs is possible with this measurement. However, the technique does not scale well below this limit[1].

LDRD project accomplishments

This LDRD started in the fall of 1997 and was quite productive in its first year. The streak tube for the 500 fs streak camera was purchased and several experiments were done on the SPA to determine the optimum components for the rest of the camera. At the same time, ongoing experiments validated the use of BPMs to measure the second moment of the beam and improved our understanding of the beam transport in the machine. In turn, this led to the development of a novel emittance measurement[2, 3, 4] [19,20].

During the second year of the LDRD progress was substantially hampered by facility upgrades in the building that houses the SPA and the departure of the individual responsible for the construction of the streak camera. Because of the facility upgrades, we were unable to produce beam in the accelerator for part of fiscal year 1997 and most of fiscal year 1998. However, during this down time we did significantly upgrade the SPA beam line and data acquisition system. We approached engineers at Bechtel Corporation to build the streak camera components that we had originally intended to make ourselves. We also made arrangements to perform Smith-Purcell experiments at Brookhaven National Laboratory (BNL).

Our second year publications consisted of two theoretical papers: one on understanding correlated transverse plasma oscillations on the root-mean-square (rms) beam moments[5] and one on the effect of diagonal correlations on the electron beam's second moments[6].

In the third and final year of this LDRD the Smith-Purcell experiment at BNL was bumped by other programmatic activities at that laboratory. We also determined that we could not afford to complete the 500 fs streak camera. As an alternative, arrangements were made to borrow a 250 fs streak camera from the Stanford Linear Accelerator (SLAC) facility. However, SLAC could not let us have the camera until October of 1999, after our LDRD funding ran out. Also, an unexpected DOE safety inspection during the time period the camera was to be at LANL made installing and using the camera impractical. We gave an invited paper at the 1999 Particle Accelerator Conference[7] based on the spin-off emittance measurement technique[3]. Also, we successfully demonstrated measuring the bunch-length of the electron beam using the transversely deflecting RF field and BPM. Preliminary data from those experiments is shown in Figure 7. A paper based on this experiment is being prepared for publication.

Overall this LDRD was a success. There were some unexpected obstacles that prevented us from accomplishing all that we set out to do, but significant progress was made none-the-less. We successfully demonstrated one of the two novel bunch-length measurements originally proposed and demonstrated a novel emittance measurement that was not part of the original proposal.

Publications

1. Russell, S. J., “Measuring the RMS Length of Short Electron Pulses with an RF Cavity and a Beam Position Monitor,” *Proceedings of the 1997 Particle Accelerator Conference*, IEEE Catalog No. 97CH36167, p2005 (IEEE, New York, 1998).
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7. Russell, S. J. and Carlsten, B. E., “Measuring Emittance Growth due to Magnetic Bunching of an Electron Beam Using the Second Moment of its Image Charge,” *Proceedings of the 1999 Particle Accelerator Conference*, IEEE catalog No. 99CH36366, p. 477 (IEEE, New York, 1999).

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Figure 1: Diagram illustrating Smith-Purcell radiation (SPR). It also defines the coordinate system and angles that define the direction of the SPR.

Figure 2: Angular dispersion relation of Smith-Purcell radiation for a grating period of 1 mm, diffraction order 1 and beam energy 8 MeV.

Figure 3: Smith-Purcell radiation intensity versus angle θ in the x plane ($\phi = 0$) for electron beam bunch-lengths of 2 ps, 1 ps and ½ ps. Grating period, d, is 1 mm, dispersion order, n, is 1 and beam height, h, is 0.5 mm.

Figure 4: Smith-Purcell radiation intensity versus angle θ in the x plane ($\phi = 0$) for electron beam bunch-lengths of 200 fs, 100 fs and 50 fs. Grating period, d, is 0.1 mm, dispersion order, n, is 1 and beam height, h, is 0.1 mm.

Figure 5: Cross section of beam position monitor showing beam quantities that can be measured. The direction of beam travel is out of the page.

Figure 6: Diagram of transversely deflecting RF field and beam position monitor bunch-length diagnostic.

Figure 7: Plot of bunch-length of the Sub-Picosecond Accelerator electron versus phase for a fixed chicane angle as measured with a transversely deflecting RF field and a beam position monitor. The bunch-length is expected to start at about 20 ps (0 degrees phase) and decrease as the phase decreases. The minimum bunch-length is expected to occur at about –39 degrees.

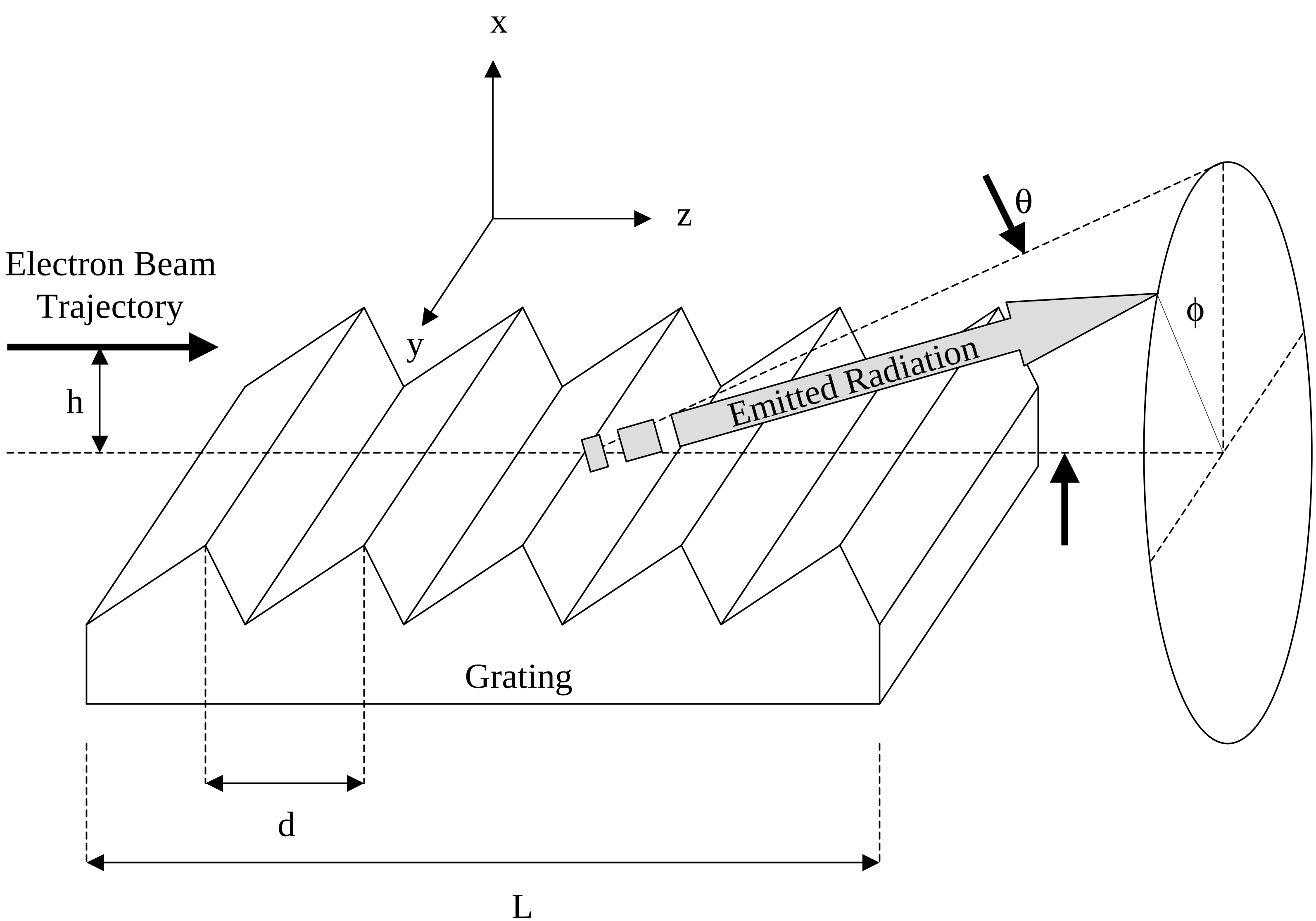
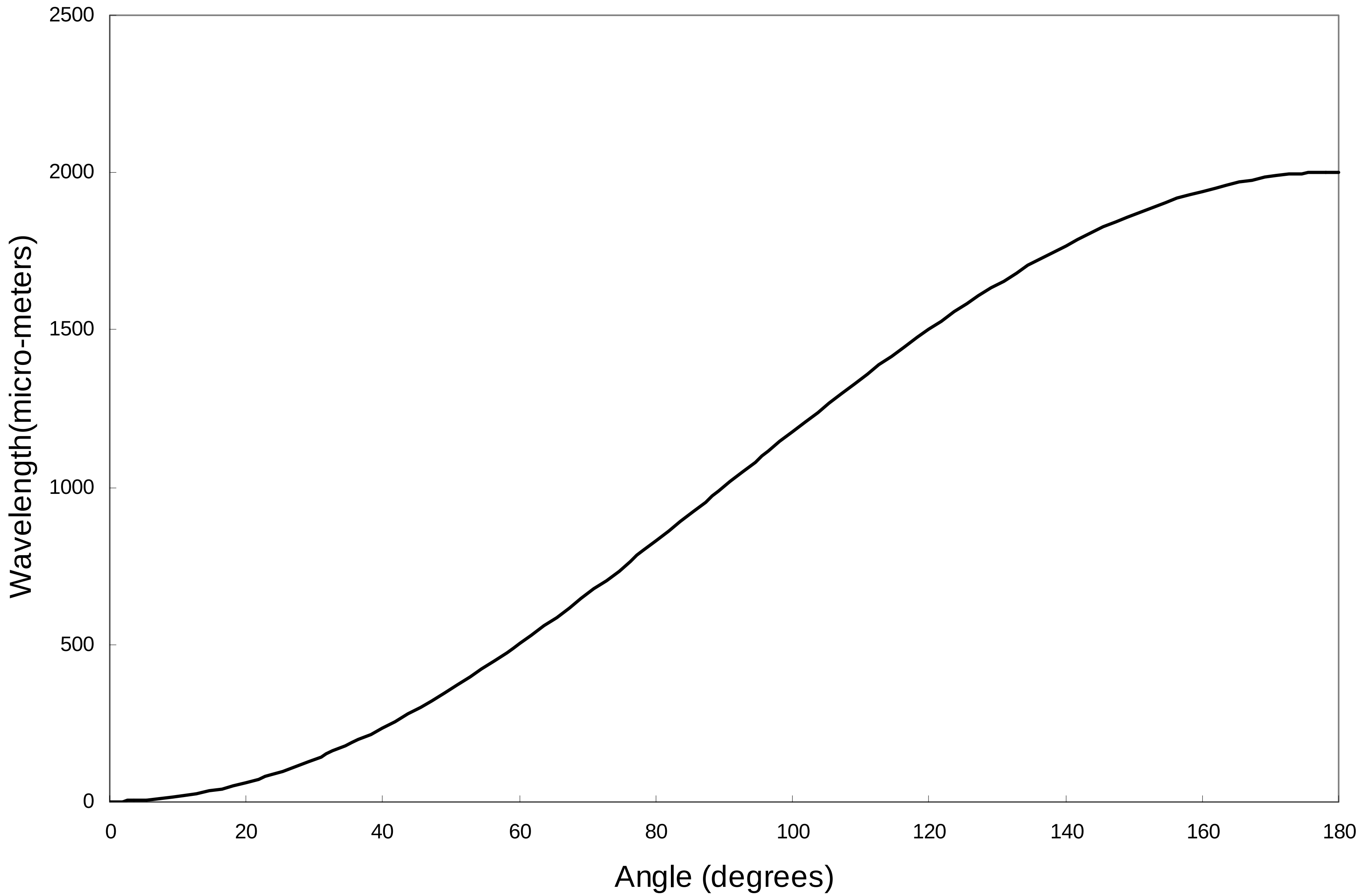
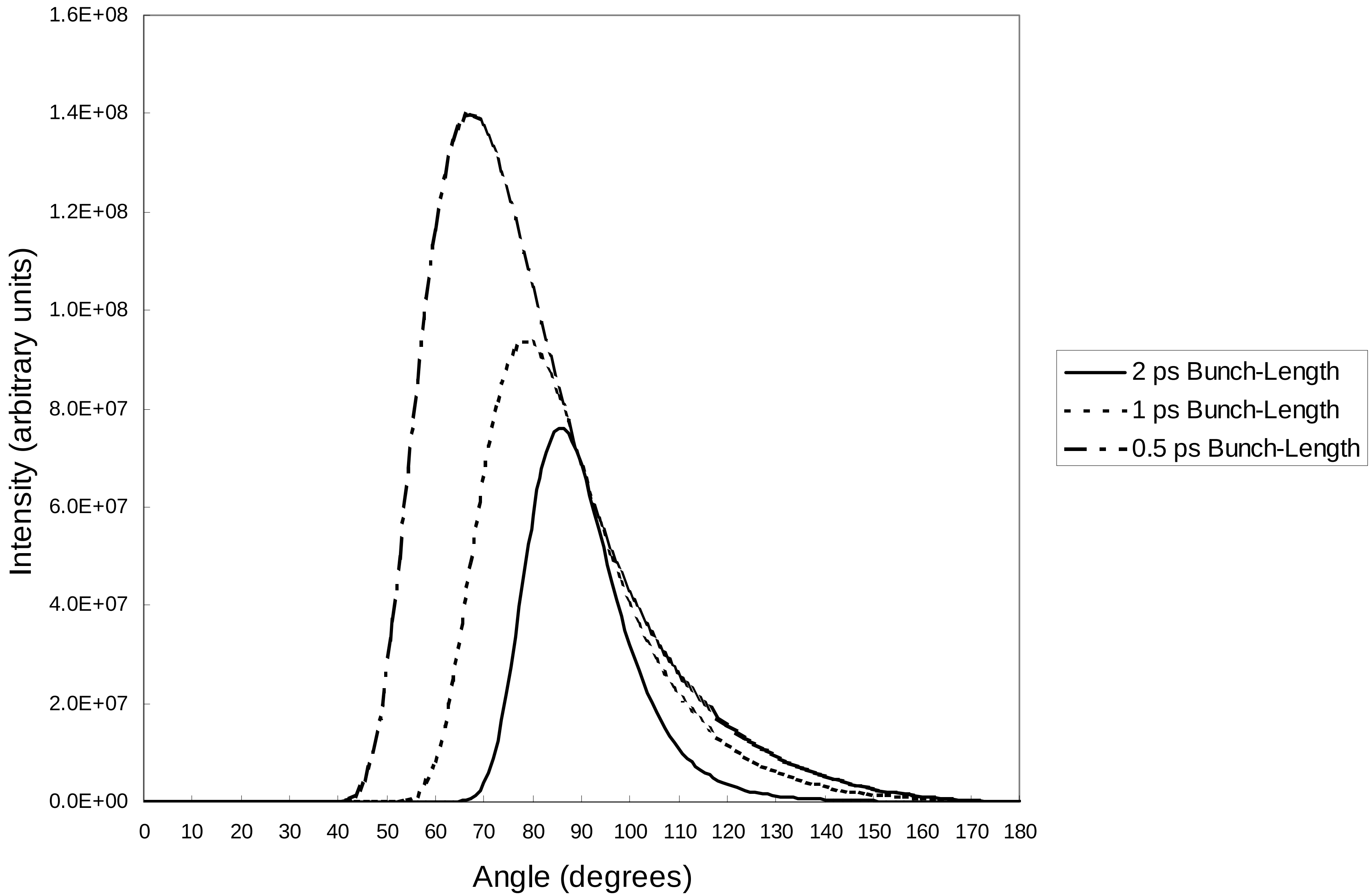

x
z
y
Electron Beam
Trajectory
h
θ
φ
Emitted Radiation
Grating
d
L

Figure 1



2500
2000
1500
1000
500
0
0
20
40
60
80
100
120
140
160
180
Wavelength(micro-meters)
Angle (degrees)

Figure 2



1.6E+08
1.4E+08
1.2E+08
1.0E+08
8.0E+07
6.0E+07
4.0E+07
2.0E+07
0.0E+00
Intensity (arbitrary units)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
Angle (degrees)
2 ps Bunch-Length
1 ps Bunch-Length
0.5 ps Bunch-Length

Figure 3

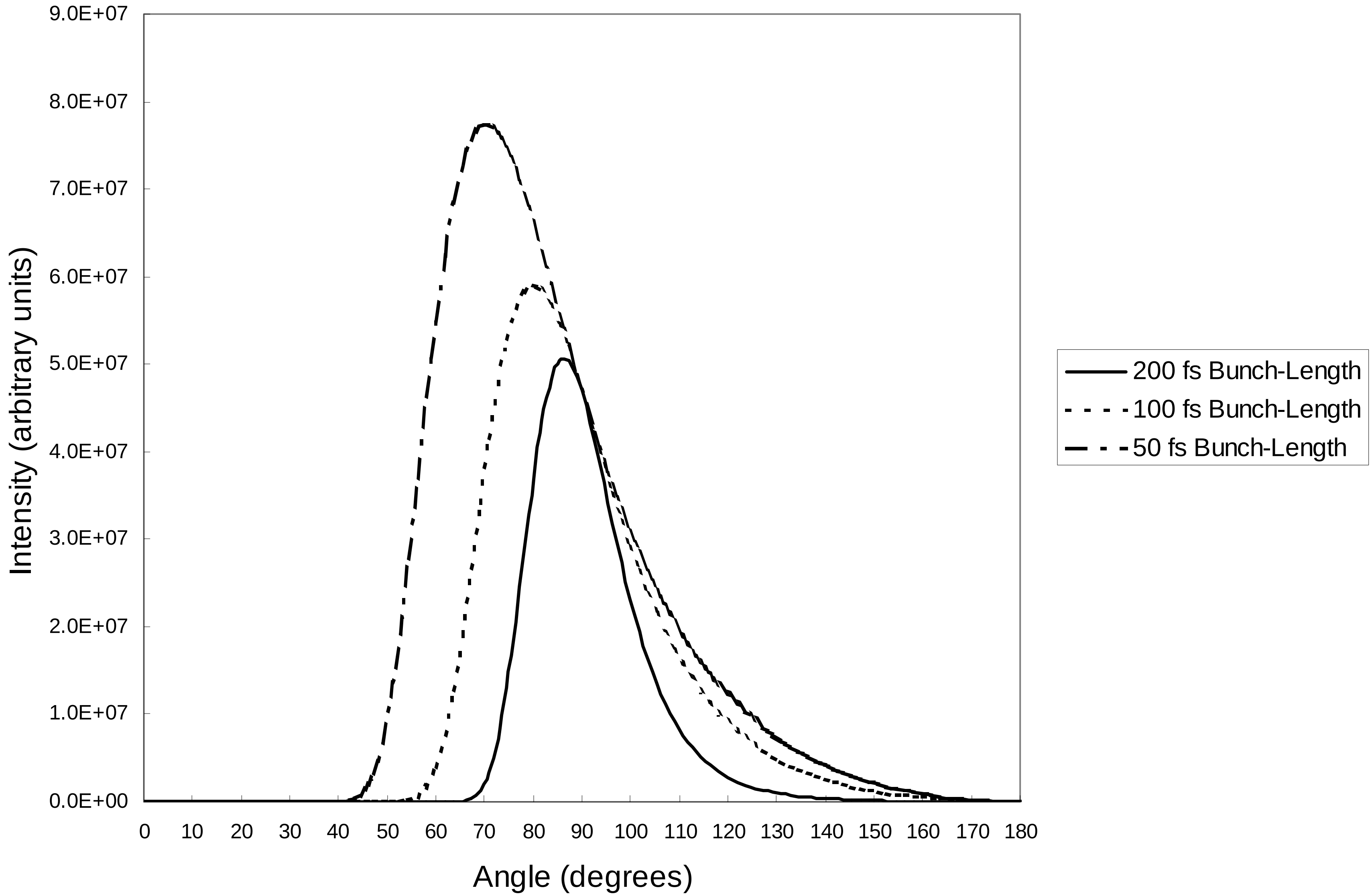
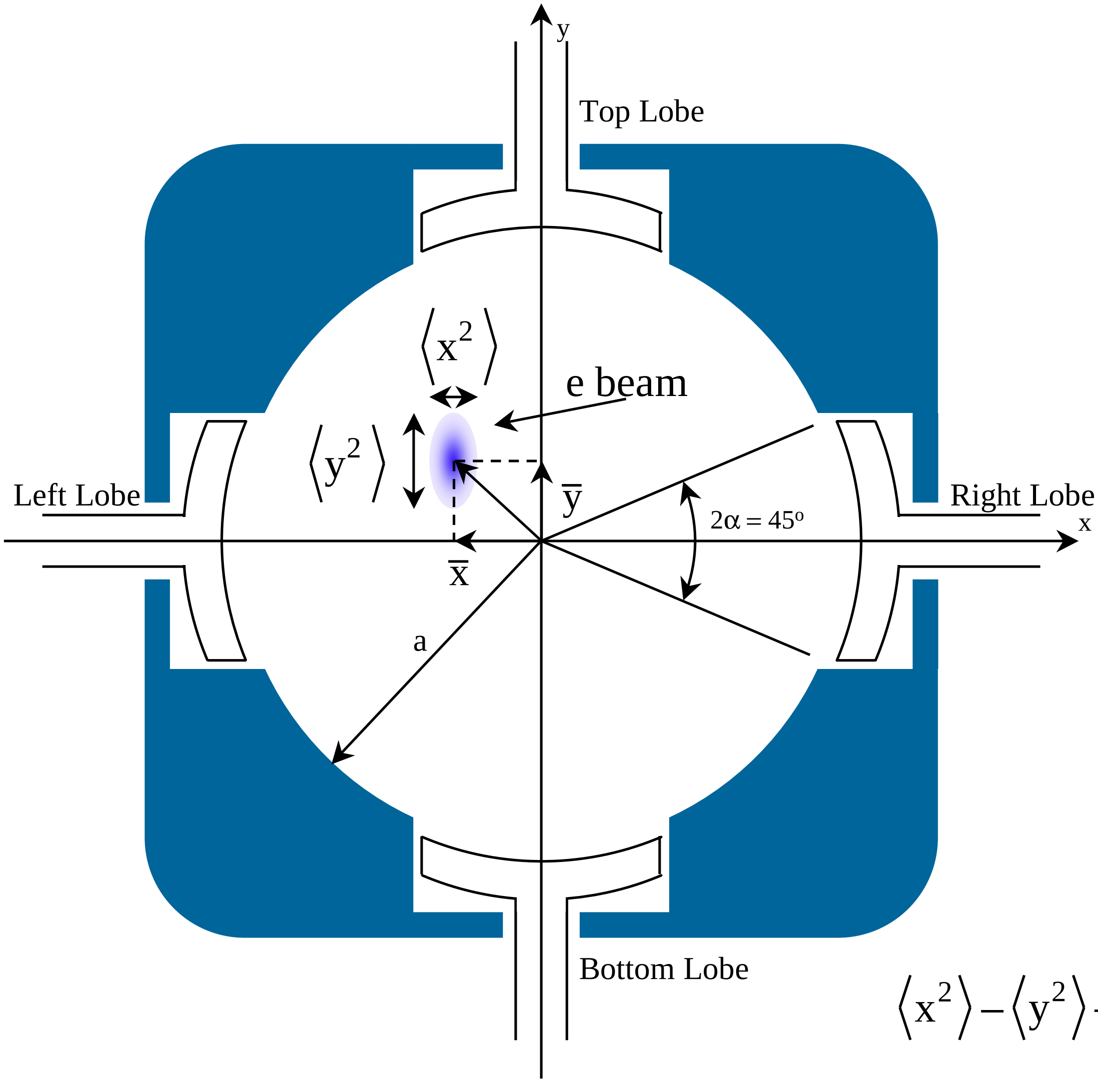

9.0E+07
8.0E+07
7.0E+07
6.0E+07
5.0E+07
4.0E+07
3.0E+07
2.0E+07
1.0E+07
0.0E+00
Intensity (arbitrary units)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
Angle (degrees)
200 fs Bunch-Length
100 fs Bunch-Length
50 fs Bunch-Length

Figure 4



- The amplitudes of the signals from the four lobes are: A_R, A_L, A_T and A_B.

- The beam intensity is given by the sum of the amplitudes.

- It can be shown that the beam center position is given by

$$\bar{x} = a \frac{\alpha}{2 \sin \alpha} \frac{A_R - A_L}{A_R + A_L}$$

$$\bar{y} = a \frac{\alpha}{2 \sin \alpha} \frac{A_T - A_B}{A_T + A_B}$$

- The second moment, or quadrupole moment of the beam is given by

$$\langle x^2 \rangle - \langle y^2 \rangle + \bar{x}^2 - \bar{y}^2 = a^2 \frac{\alpha}{\sin 2\alpha} \frac{A_R + A_L - A_T - A_B}{A_R + A_L + A_T + A_B}$$

Figure 5

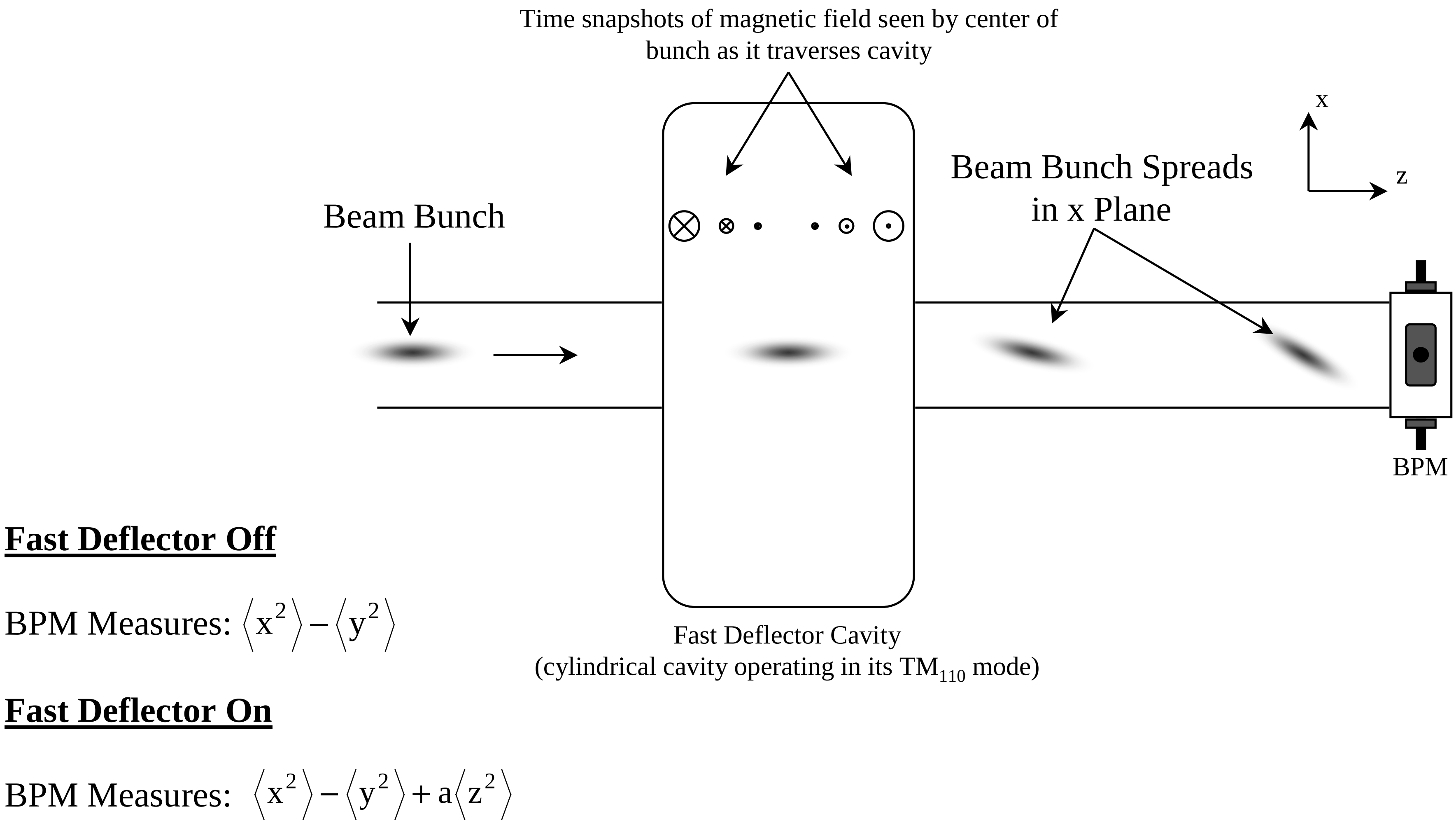
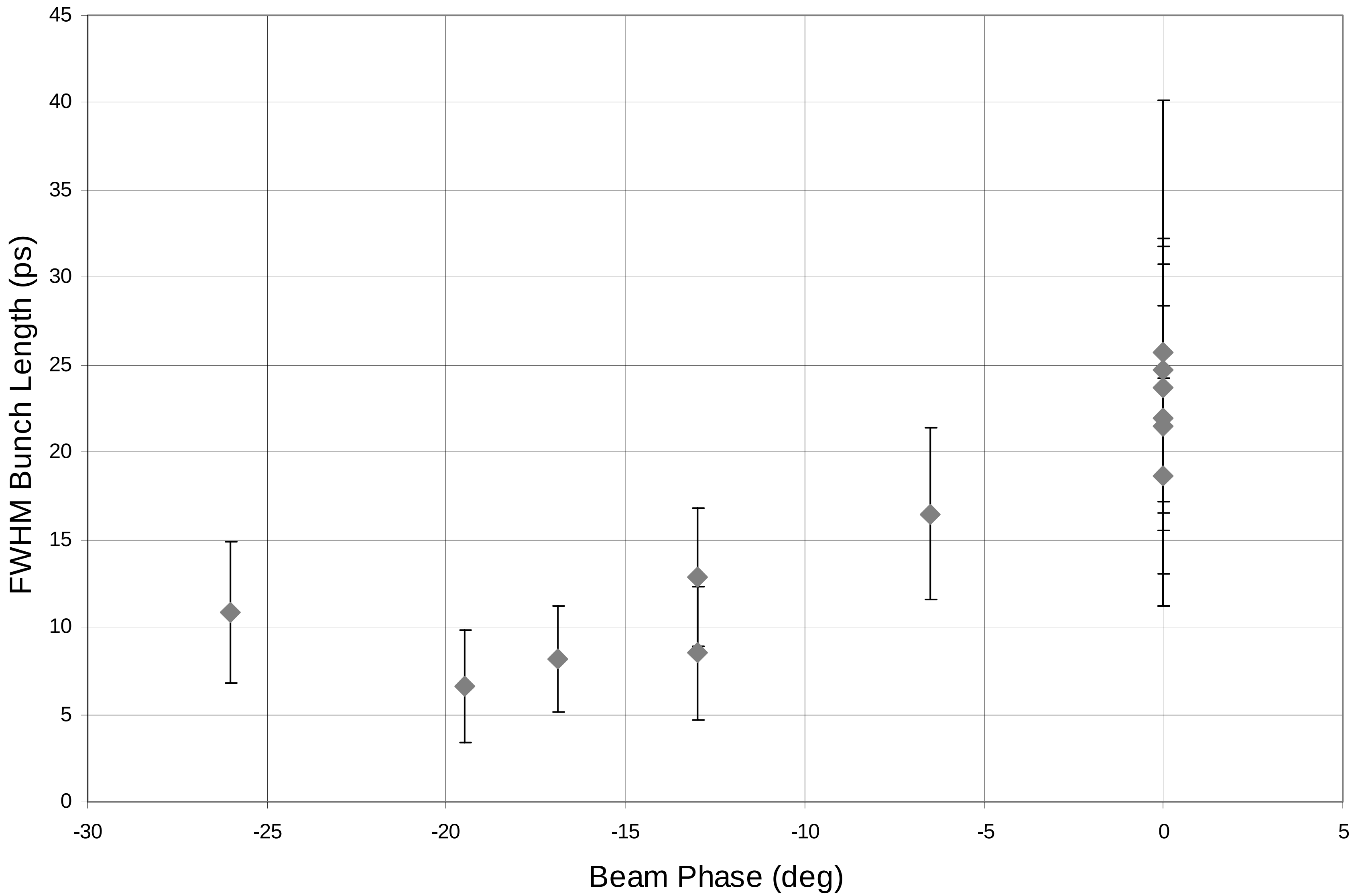

Time snapshots of magnetic field seen by center of bunch as it traverses cavity
x
z
Beam Bunch
Beam Bunch Spreads in x Plane
BPM
Fast Deflector Cavity
(cylindrical cavity operating in its TM_{110} mode)
Fast Deflector Off
BPM Measures: $\langle x^2 \rangle - \langle y^2 \rangle$
Fast Deflector On
BPM Measures: $\langle x^2 \rangle - \langle y^2 \rangle + a\langle z^2 \rangle$

Figure 6



FWHM Bunch Length (ps)
45
40
35
30
25
20
15
10
5
0
-30
-25
-20
-15
-10
-5
0
5
Beam Phase (deg)

Figure 7